Mat 320 - Stout Name:____________________

QUIZ 8: ABSTRACT ALGEBRA

Let G be a group and H be a subgroup of G. We define $gH = \{gx \mid x \in H\}$ to be a left coset of H and $Hg = \{xg \mid x \in H\}$ to be a right coset. The left cosets (or the right cosets) always partition the group G into $[G : H]$ equal parts. The sugroup H is said to be normal if $gH = Hg$ for all $g \in G$.

Problem 1: Let $H < G$ be an arbitrary subgroup of G (not necessarily normal). Prove that if $g \in H$, then $gH = H$ and $Hg = H$.

Problem 2: Prove that H is a normal subgroup of G if and only if $gHg^{-1} = H$ for all $g \in G$.

Problem 3 Let H and K be subgroups of G and define the double coset of H and K by

$$HgK = \{hgk \mid h \in H, k \in K\}$$

where $g \in G$. Show that if g is in H (or is in K) and either H or K is normal, then the double coset[1] in this case is actually a subgroup S of G. In addition, show that if H is normal then any double coset is actually a left coset of this subgroup S.

[1]Note that if H is not normal, then the size of a double coset doesn't necessarily divide the order of the group and the size of two double cosets are not necessarily the same.