Mat 320 - Stout Name:______________________

QUIZ 6: ABSTRACT ALGEBRA

Problem 1: The Quaternion group Q_8 is given by the presentation on three generators i, j, and k with $i^2 = j^2 = k^2 = ijk$ where ijk is a flip (element of order 2). If we refer to the element ijk by -1, we can write the presentation as

$$Q_8 = \langle i, j, k \mid i^2 = j^2 = k^2 = ijk = -1,\ (-1)^2 = 1\rangle$$

Use the identities above, to show the following (Hint: In both Part B and Part C below, take $ijk = -1$ and multiply on right by the correct elements.)

Part A. For any x in Q_8, $-1 \cdot x = x \cdot -1$. For this it is enough to argue for the element i as the proof will be the same for all other elements. Then, note

$$ijk = -1$$

(1) $$-jk = i \cdot (-1) \qquad \text{(multiply on the left by i)}$$

$$-1(i) = i \cdot (-1) \qquad \text{(You will essentially prove this step below)}$$

Important Note: Strictly speaking the direction of these proofs are circular. We should first develop the Cayley table below and then argue that -1 commutes with every element. But, I want you to use this fact first to make the proofs below easy.

Part B. $ij = k$

Part C. $kji = -1$

Part D. $jij = i$

Part E. $ij = -ji$

Problem 2: Fill in the remaining entries of the Cayley table for the Quaternion group Q_8 below

$*$	1	-1	i	$-i$	j	$-j$	k	$-k$
1	1	-1	i	$-i$	j	$-j$	k	$-k$
-1	-1	1	$-i$	i	$-j$	j	$-k$	k
i	i	$-i$	-1	1	k			
$-i$	$-i$	i	1	-1				
j	j	$-j$	$-k$		-1		i	
$-j$	$-j$	j				-1		
k	k	$-k$			$-i$		-1	
$-k$	$-k$	k						-1

Problem 3. Find all the cyclic subgroups of Q_8. <u>Hint</u>: There are exactly five of them.