Mat 320 - Stout Name:____________________

QUIZ 5: ABSTRACT ALGEBRA

Problem 1: We say that two integers a and b are relatively prime if the greatest common divisor is 1 – i.e., $GCD(a, b) = 1$ were GCD denotes the largest natural number which divides both a and b.

Part A. Show that for any prime number p and any $1 \leq m < p$, $GCD(p, m) = 1$.

Part B. Find all $1 \leq m < 6$ such that m is relatively prime to 6.

Part C. Find all $1 \leq m < 12$ such that m is relatively prime to 12.

Problem 2: Let $G = \langle g \rangle$ be a cyclic group of order n, where g is a generator of G – this means that n is the smallest natural number such that $g^n = 1$.

Part A. Show that if k is relatively prime to n, then g^k is also a generator of G. *Hint: Prove by contradiction by letting* $(g^k)^a = 1$ *for some* $a < n$ *and use the fact that* $g^b = 1$ *implies* n *divides* b *since* g *is a generator of a cyclic group.*

Part B. Assuming d divides n (i.e., $n = m \cdot d$), consider the element g^d. Describe the subgroup $\langle g^d \rangle$ and determine its order.

Part C. Use Part B of Problem 1 and the previous two parts of this problem to determine all possible generators g^k of the cyclic group C_6 of order 6 and all possible proper subgroups of the C_6 given by $\langle g^d \rangle$.