Mat 320 - Stout   Name:______________________

QUIZ 4: ABSTRACT ALGEBRA

Problem 1. Complete the following definition.

DEFINITION 0.1. We say that a tuple $(G, \cdot)$ is a **group** if it satisfies the following four conditions (sometimes referred to as the Axioms of Group Theory).

1. Closure: For any g and h in G, $g \cdot h$ is also in G.

2. Associativity: For all g, h, f in G, we have $g \cdot (h \cdot f) = \underbrace{\qquad\qquad}_{\text{put answer here}}$

3. Identity: There exists an element e in G such that for all g in G, $\underbrace{\qquad\qquad\qquad\quad}_{\text{put answer here}}$

4. Inverses: For all g in G, there exists $\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{put answer here}}$

Problem 2. Refer to the conditions above as Axiom 1, Axiom 2, Axiom 3, and Axiom 4. Complete the following proof of the left cancellation law making sure to refer to the appropriate Axiom for each step you write down.

THEOREM 0.2 (LCL). *Let $(G, \cdot)$ be a group. Then, for all g, x, y in G,*

$$g \cdot x = g \cdot y \implies x = y.$$

PROOF. Let g, x, and y be arbitrary elements of G. Assume further that $g \cdot x = g \cdot y$. By Axiom 4 (Inverses), there exists an element *...complete the proof in the space below by following directions above...*

$\square$