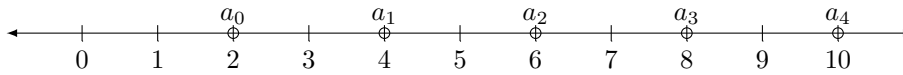


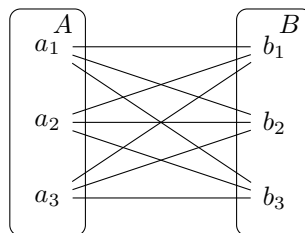
QUIZ 2: ABSTRACT ALGEBRA

Example of arithmetic progression with difference 2 and initial term 2

Problem 1. An *arithmetic progression* of length k with difference q and initial term a_0 is a set of the form

$$\{x \in \mathbb{Z} \mid x = a_0 + mq \text{ s.t. } m = 0, 1, 2, 3, \dots, k-1\} \subset \mathbb{Z}.$$

- Describe the arithmetic progression S of length 10 with difference 3 and initial term 1 as a set $S = \{ \quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad, \quad \}$
- Is it true that $7 \in S$? *yes or no*
- Is it true that $20 \notin S$? *yes or no*
- For *extra-credit*, describe the set obtained by letting $k \rightarrow \infty$ but leaving the difference and initial term the same.



Visualization of a Cartesian Product of two arbitrary sets A and B .

Problem 2. Let A and B be two sets and form the cartesian product

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

- Is it true that $A \times B = B \times A$? *yes or no*
- Consider $p : A \times B \rightarrow A, (a, b) \mapsto a$. Is this map p surjective? *Explain.*
- Construct an injective map $\iota : A \rightarrow A \times B$ by assign a value $a \mapsto \iota(a)$.
- For *extra-credit*, prove that the map you construct in item c is injective – i.e., show that $\iota(a_1) = \iota(a_2) \implies a_1 = a_2$.