Mat 320 - Stout Name:______________________

QUIZ 12: ABSTRACT ALGEBRA

Problem 1: Consider $\mathbb{H}$ to be the group ring $\mathbb{R}[Q_8]$ defined[1] to be all elements of the form

$$x_1 \cdot 1 + x_2 \cdot (-1) + x_3 \cdot i + x_4 \cdot (-i) + x_5 \cdot j + x_6 \cdot (-j) + x_7 \cdot k + x_8 \cdot (-k)$$

with x_i elements of $\mathbb{R}$ for $i = 1, \ldots, 8$.

Part A. Create ring homorphisms $\mathbb{R} \to \mathbb{H}$ and $\mathbb{H} \to \mathbb{R}$.

Part B. Is $\mathbb{C}$ a subring of $\mathbb{H}$? Why or why not?

Part C. Consider the ideal $I = \langle i \rangle := \{r \cdot i \mid r \in R\} \trianglelefteq R$. Show that I is a non-proper ideal – i.e. show that $I = \mathbb{H}$.

Problem 2: Let I and J be two left-ideals of a ring R. Define $I + J$ to be the subset of R given by

$$I + J := \{f + g \mid f \in I,\ g \in I\}$$

Show that $I + J$ is also an ideal of R – i.e., show that $I + J$ is a subgroup of $(R, +)$ and that $I + J$ has the left absorption property.

[1]The non-commutative division ring $\mathbb{H}$ is known as the Quaternions and is the simplest model of such an algebraic structure. Weddererburg's Little Theorem states that every finite division ring D is commutative – i.e., D is a field in that case.