Mat 320 - Stout Name:____________________

QUIZ 11: ABSTRACT ALGEBRA

Problem 1: Describe the group $3\mathbb{Z}/6\mathbb{Z}$ as a set and show that it is a subgroup of $\mathbb{Z}/6\mathbb{Z}$. Then, apply the third isomorphism theorem to $(\mathbb{Z}/6\mathbb{Z})/(3\mathbb{Z}/6\mathbb{Z})$.

Problem 2: Let $\mathbf{GL}_2(\mathbb{R})$ denote all 2×2 non-singular matrices with real entries and let $\mathbf{SL}_2(\mathbb{R}) \subset \mathbf{GL}_2(\mathbb{R})$ denote all such matrices with determinant 1. Use the first isomorphism theorem to find a group G which is isomorphic to $\mathbf{GL}_2(\mathbb{R})/\mathbf{SL}_2(\mathbb{R})$ by finding a surjective group homomorphism $\mathbf{GL}_2(\mathbb{R})$ onto some other group G whose kernel is $\mathbf{SL}_2(\mathbb{R})$.

Problem 3: Let C_n denote the cyclic group of order n. And, let D_n be the dihedral group of order $2n$. Construct a surjective group homomorphism

$$f : D_n \to C_2$$

and show that it is indeed a surjective group homomorphism. Apply the first isomorphism theorem to group homomorphism.

Problem 4: Let Q_8 denote the quaternion group. Find a natural sequence[1] of group homomorphisms

$$1 \to \mathbb{Z}/2\mathbb{Z} \xrightarrow{\iota} Q_8 \xrightarrow{f} \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \to 1$$

where ι is injective and $\iota(\mathbb{Z}/2\mathbb{Z})$ is normal and f is surjective.

[1] Note that $C_n \xrightarrow{\sim} \mathbb{Z}/n\mathbb{Z}$ and that the Klein four group V is isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.