Mat 320 - Stout Name:______________________

QUIZ 10: ABSTRACT ALGEBRA

Problem 1: Consider the general question on the group action below.

Part A. Let G be a group acting on itself by left multiplication. Calculate the orbit of the identity element e.

Part B. Let G be a group acting on itself by conjugation. Calculate the orbit of the identity element e.

Part C. Assume that the stabilizer subgroup G_x of a group action $G \curvearrowright X$ is trivial – i.e., $G_x = \{e\}$. What can we say about the orbit O_x of x?

Problem 2 Consider the group actions below.

Part A. Let $C_n = \langle x \rangle$ be the cyclic group of order n. Definite the action of C_n on the set $X = \{1, 2, 3, \ldots, n\}$ by $x^k \cdot i = i + k \bmod n$. Show that this action is transitive.

Part B. Let $X = \{1, 2, 3, \ldots, n\}$ and let $G = \mathrm{Sym}(X) = S_n$. Does G act transitively on X?

Problem 3. Give an non-trivial example of each.

Part A. A injective group homomorphism between two finite groups.

Part B. A surjective group homomorphism between two finite groups.

Part C. A surjective group homomorphism from an infinite group onto a finite group.

Part D. A non-trivial group automorphism of $\mathbb{R}$ and a non-trivial group automorphism of $\mathbb{C}$ which leaves $\mathbb{R}$ fixed.