MAT 301. FIRST AND SECOND DERIVATIVE TESTS: PRACTICE PROBLEMS.

In each of the following problems you will get *extra credit* if you offer a detailed sketch of the funciton $f(x)$.

Problem 1. Consider the function $f(x) = x^3 + x^2 - x$ on the interval $[-2, 4]$. Find the two critical values and the one possible inflection point of $f(x)$.

Problem 2. Consider the function $f(x) = x^2 e^{-3x}$ on the interval $[-1, 1]$. How many critical points does $f(x)$ have in the interval $(-1, 1)$? Find the global maximum and minimum of the function $f(x)$ on $[-1, 1]$. Hint: You do not need to use the 1st or 2nd derivative test for this problem (although you can if you would like).

Problem 3. Consider the function $f(x) = \frac{x^2}{x^2-1}$. By applying the quotient rule, the 1st derivative of this function is $f'(x) = \frac{-2x}{(x^2-1)^2}$. Use the 1st derivative test to find the local maximum and minimums of $f(x)$. Hint: There is only one of them.

Problem 4. The second derivative of the function $f(x)$ in Problem 3 is $f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3}$. Use this second derivative to apply the 2nd derivative test of the function $f(x)$. Does your answer agree with your answer in Problem 3?

Problem 5. Given the second derivative $f''(x)$ in Problem 4 above, solve the equation $f''(x) = 0$ to find potential inflection points and look at the sign changes of $f''(x)$ around them to see if they are truly inflection points.

Problem 6. Calculate the second derivative of the function $f(x) = \frac{x^2-2x+4}{x-2}$ and use this to show that it has $f(x)$ has no inflection points.

Problem 7. Consider the function $f(x) = \frac{(x-1)^2}{x^2+3}$ defined for all real numbers. Use the first derivative test to find the local maximums and minimums. Hint: There is only one of each.

Problem 8. Consider the function $f(x) = x^{2/3} - \frac{9x}{4}$ defined for all real numbers (note that $\sqrt[3]{-a} = -\sqrt[3]{a}$ since $\sqrt[3]{-1} = -1$ because $(-1)^3 = -1$). Use the 2nd derivative test, to find the local maximums and minimums of this function. Hint: Just apply the power rule twice using fractional exponents.

Problem 9. Consider the function $f(x) = x^{4/5}(x-1)$ defined for all real numbers. Use the 1st derivative test to find the local maximums and minimums. Hint: Note that one critical value is given by $x = 0$ because the first derivative $f'(x)$ is not defined there.

Problem 10. Consider the function $f(x) = \sin(x) + \cos(x)$ defined on the interval $[0, 2\pi]$. Use the 1st derivative test to find the local maximums and minimums of this function.